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THE DEPENDENCE OF HOT-WIRE CALIBRATION ON GAS TEMPERATURE AT LOW REYNOLDS NUMBERS

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Abstract—Heat-transfer measurements from a hot-wire placed perpendicular to a horizontal air stream, have been made with a constant-temperature anemometer for $0.03 < Re < 0.80$. Gas temperatures were varied between 283 and 353 K at a constant wire temperature of 473 K. After correction for heat conduction to the wire supports the experimental results could be represented by the relation

$$Nu_m = a_m + b_m Re^m$$

While a_m and b_m are approximately constant, m is found to vary with gas temperature in the same manner as the thermal conductivity of the gas. Gas thermal conductivity and kinematic viscosity are taken at the gas temperature and not at the film temperature, as is common practice.

NOMENCLATURE

a, b, numerical constants for continuum Nu;
a_m, b_m, numerical constants for non-continuum Nu;
A, hot-wire parameter [A^2];
B, hot-wire parameter [$A^2 \cdot m^{-m} \cdot s^m$];
d, hot-wire diameter [m];
Kn, Knudsen number;
l, hot-wire length [m];
m, exponent;
n, exponent;
Nu, Nusselt number;
Pr, Prandtl number;
r, radius [m];
R, resistance of hot wire [Ω];
Re, Reynolds number;
T, absolute temperature [K];
U, gas velocity [m/s];
V, hot-wire tension [V].

Greek symbols

β, temperature coefficient of wire resistance [K^{-1}];
λ, thermal conductivity [W/(m·K)];
ν, kinematic viscosity [m^2/s];
ρ, gas density [kg/m^3];
Φ, power [W].

Subscripts

a, axial;
c, continuum;
f, film temperature $T_f = \frac{1}{2}(T_w + T_g)$;
g, gas;
m, value derived from measurement;
nc, non-continuum;
0, at reference temperature $T_0 = 273$ or 293 K;
w, wire.

1. INTRODUCTION

THE CALIBRATION of hot-wire anemometers is usually based on some relation expressing the dependence of the Nusselt number on the Reynolds and Prandtl numbers. Various relations of this kind are given in literature, e.g. [1–5]. However, no unique relation exists. Factors that can effect the form of the relation are: (i) the ratio of the diameter, d, to the length, l, of the wire, determining *inter alia* the influence of heat conduction to the supports; (ii) the Knudsen number, being the ratio of the mean free path in the gas to the wire diameter; (iii) the range of Re- and Pr-numbers; and (iv) the wire and gas temperatures.

Usually the physical properties of the gas, viz. λ, ν and ρ, are taken at the film temperature, but some authors recommend a different procedure. The temperature dependence of the calibration curve is usually found from the variation of these properties with temperature (cf. [2]); sometimes a separate temperature "loading factor" is introduced [3].

For hot wire investigations of turbulent quantities under non-isothermal conditions, cf. Blom [6–7], it is essential to know the temperature dependence of the hot-wire calibration curve. The degree of accuracy must be high, because the derivative of the curve with respect to the flow velocity must also be known accurately, to determine the sensitivity of the hot-wire to velocity and temperature variations, Elsner [8] found experimentally different values for the sensitivity with respect to velocity variations than the ones derived from the relations of Kramers [2] and Collis and Williams [3].

In previous investigations more attention has been paid to the influence of the wire temperature than to that of the gas temperature. Therefore we determined the calibration curves of a constant-temperature hot-wire anemometer in air flow at different gas temperatures in the temperature range of interest in our investigations. The results are presented in this paper. They are compared with those of other investigators and an attempt is made to interpret them on the basis of a Nu–Re relationship.

2. SEMI-EMPIRICAL RELATIONS

For air the Prandtl number can be considered as fixed and equal to 0.71 in the temperature range under consideration; hence in this case Nu depends on Re only. The relation between these two quantities for Re between 0.01 and 40 is often written in the form

$$Nu_c = a + bRe^n \tag{1}$$

with constant a, b and n. The relation is supposed to be universal for continuum flow perpendicular to the axis of infinity long circular cylinders. Gas properties are evaluated at the film temperature,

$$T_f = \tfrac{1}{2}(T_w + T_g).$$

Values of a, b and n, found by a number of authors are given in Table 1. It should be noted that Collis

Table 1. Values of parameters in equation (1) given by various authors

Reference	a	b	n	Range of Re
King [1]	0.32	0.69	0.50	0.55 to 55
Kramers [2]	0.39	0.51	0.50	10^{-2} to 10^4
Collis and Williams [3]	0.24	0.56	0.45	0.02 to 44
Andrews *et al.* [4]	0.34	0.65	0.45	0.015 to 20

and Williams suggested to multiply Nu_c in (1) by a "loading factor" $(T_g/T_f)^{0.17}$ to compensate for different wire temperatures. The influence of the temperature jump at the wire, as given in the literature [3–4], is expressed as:

$$Nu_c = \frac{Nu_{nc}}{1 - 2Kn \cdot Nu_{nc}} \tag{2}$$

where Nu_{nc} is the non-continuum value of Nu, whilst it is assumed that the thermal accommodation coefficient equals 0.9. For air at a wire temperature of 473 K and atmospheric pressure the mean free path is about 0.14 μm; thus for a wire diameter of 2.5 μm or higher $Kn < 0.045$. For $Nu_{nc} < 1$ the correction is therefore less than 9%. Because the correction increases with increasing values of Nu it will affect the values of a, b and n.

In practice the electric tension over the wire, V, is measured, from which the total power input, $\Phi_{tot} = V^2/R_w$, follows. This quantity has to be corrected for the heat transferred by conduction to the supports. One has (cf. Hinze [2])

$$\Phi_{tot} = \Phi_{conv} + \Phi_{cond}. \tag{3}$$

where the subscripts refer to convection and conduction, and

$$\Phi_{cond}/\Phi_{conv} = \frac{d}{l}\left(\frac{R_w}{R_g}\frac{\lambda_w}{\lambda_g}\frac{1}{Nu}\right)^{\frac{1}{2}}. \tag{4}$$

The influence of Φ_{cond} becomes negligible for sufficiently small values of d/l. Andrews *et al.* [4] found experimentally that n is independent of the heat conduction to the wire supports if $d/l < 2.5 \cdot 10^{-3}$. However, a and b depend more strongly on Φ_{cond} than n does.

In our measurements we used a short wire with a relatively large d/l ratio. Therefore the correction for heat conduction was approximately 8% and was applied in all cases. Φ_{conv} found from (4) was represented in the usual manner by

$$\Phi_{conv} = (A + BU^m)(R_w - R_g) \tag{5}$$

which corresponds with a relation

$$Nu_m = a_m + b_m Re^m \tag{6}$$

with

$$A = (\pi l/\beta R_0)\lambda_g a_m \tag{7}$$

and

$$B = (\pi l/\beta R_0)(d/\nu_g)^m \lambda_g b_m. \tag{8}$$

A and B in these equations depend on temperature through the gas properties λ_g and ν_g. For air in the range of gas temperatures under consideration one has:

$$\lambda/\lambda_0 = (T/T_0)^{0.814} \tag{9}$$

$$\nu/\nu_0 = (T/T_0)^{1.733} \tag{10}$$

For $T_0 = 273.15$ K one has $\lambda_0 = 0.0241$ W/(m·K) and $\nu_0 = 13.3 \cdot 10^{-6}$ m²/s.

3. EXPERIMENTAL ARRANGEMENT

The calibration experiments were performed in a constant-temperature room kept at 293 K. The apparatus used is sketched in Fig. 1.

The hot-wire (F) is situated behind a glass tube (E), with a precision boring, in which a Poiseuille flow is established. The hot-wire is placed normal to the tube axis, with its middle on this axis at a distance of 1 mm behind the tube. A large box (N), kept at the desired gas temperature, shields the hot wire from environmental influences.

Dry air flows from a cylinder (A) through a flow meter (D) and a heating compartment (L) to the glass tube (E). The flow rate is adjusted accurately by means of a reducing value (B) with a two-step pressure control.

The tension over the hot wire is determined from the output of an anemometer unit (I, DISA 55 D00) by means of a digital voltmeter (J, Fluke 8300A).

The air is kept at the desired temperature by thermostatic control through the thermostats K and M, connected respectively to the water mantles around the glass tube (E) and the box (N). A thermocouple (G) is placed behind the wire to check the temperature of the air after passing the wire. For the mean velocity over the wire, U_w, one finds from the Poiseuille profile

$$U_w = \frac{1}{l}\int_{-\frac{1}{2}l}^{\frac{1}{2}l} U(r)\,dr = U_a(1 - l^2/12r^2) \tag{11}$$

where r is the tube radius and U_a the axial velocity. The latter is twice the flow velocity averaged over the tube section, which is measured by the flow meter.

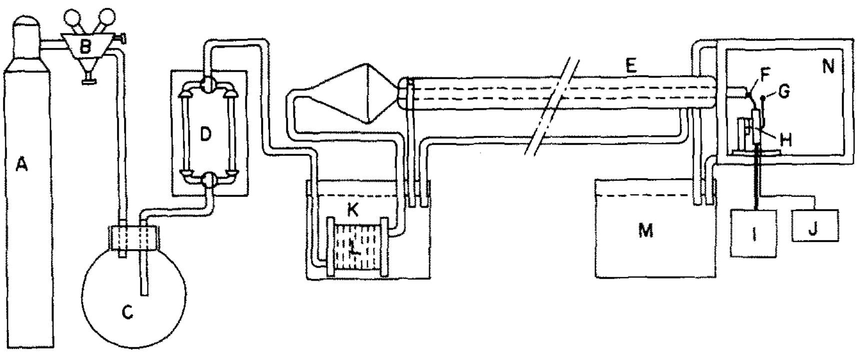


FIG. 1. Experimental set-up for calibration of hot wires at low air velocities. A—air cylinder; B—reducing valve; C—glass vessel; D—flow meter; E—glass tube; F—hot wire; G—thermocouple; H—positioning unit; I—anemometer; J—digital voltmeter; K—thermostatic water bath; L—heat exchanger; M—thermostatic water bath; N—stainless steel box with water mantle.

The wire used was of platinated tungsten; it had a diameter of 2.5 μm a length of 1.93 mm. It was welded between two steel needles, with a diameter of 0.6 mm and a length of 10 mm. These were fixed in a ceramic cylinder (diameter = 4 mm), which was held in an adjustment unit (H), allowing for an accurate positioning of the wire.

It was checked that at room temperature the flow inside the tube was of the Poiseuille type up to $U_a = 6$ m/s corresponding to $Re = U_a d/2\nu = 2 \cdot 10^3$.

The relative accuracy of the velocity measurement was 0.7%, corresponding to an absolute accuracy of 0.01 m/s. The accuracy of the measurement of R_g was 0.05%, that of R_w was 0.17%.

The relative accuracy of the measurement of V was 0.01%; thus the relative accuracy of Φ_{tot} is about 0.2%. The relative accuracy of $\Phi_{cond}/(R_w - R_g)$ is about 1%.

4. EXPERIMENTAL RESULTS

The dependence of the hot-wire tension, V, on the air velocity is shown in Fig. 2. From V^2/R_w the values of $\Phi_{cond}/(R_w - R_g)$ are found in the manner described in Section 2. From these the experimental values of A, B and m in equation (5) are determined by a method of least squares, using a digital computer program [9–10]. The program starts with an estimated value, m_0, of m, lying in the interval 0.3 to 0.6, and computes the corresponding values A_0 and B_0. After linearization around m_0, improved values of m, A and B are computed. The process is repeated until a best fit is obtained.

The measured values of U_w are found to lie well within $\pm 1\%$ of the best fitting curve. An example for $T_g = 293$ K is given in Fig. 3. Values of A, B and m, found in this way, for a wire temperature of 473 K and gas temperatures in the range 283–353 K are given

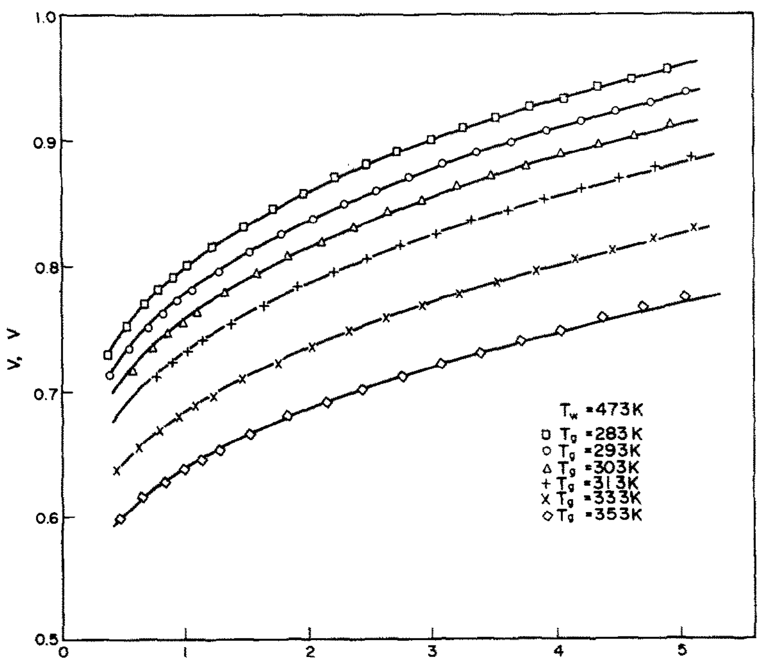


FIG. 2. Dependence of hot-wire tension, V, on air velocity, U, at different gas temperatures.

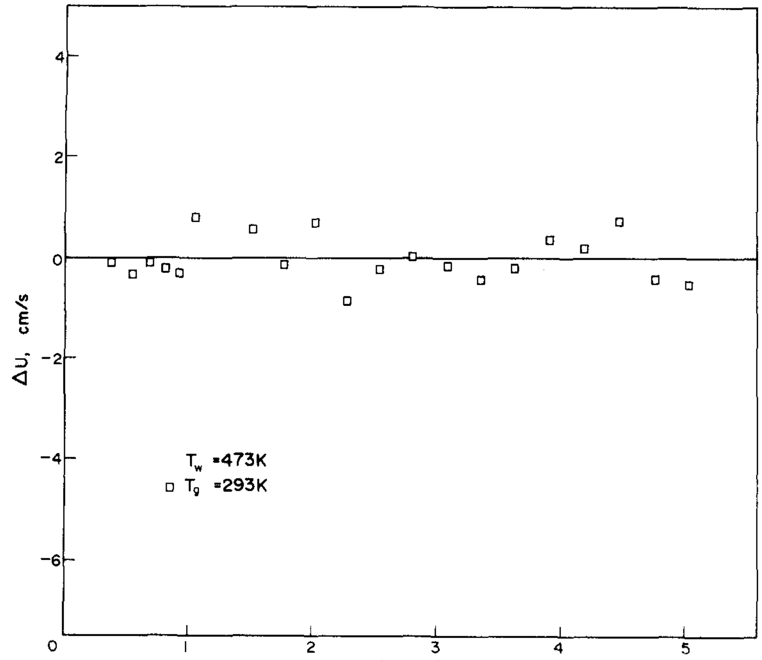


Fig. 3. Values of ΔU for gas temperature 293K with respect to the curve of best fit.

in Table 2. In addition values of AB and mB are presented. At gas temperatures of 293 and 333 K values of A and B were computed at two fixed values of m, viz. 0.40 and 0.45. The results are given in Table 3.

5. DISCUSSION

From Table 2 it can be seen that with increasing gas temperature A and m increase and B decreases. The products AB and mB are constant within the accuracy determined by experimental error. For a fixed value of m the values of A and B, giving the corresponding curve of best fit, are independent of T_g, as follows from Table 3. The product AB has about the same value as in Table 2, but mB depends on the choice of m and differs for both values of m from the values given in Table 2.

A curve of the right shape is only obtained when m is allowed to vary with temperature. This is demonstrated in Fig. 4 for $m = 0.40$ and in Fig. 5 for $m = 0.45$. Although in both cases relative deviations, viz. $\Delta U/U$, are less than 2%, their systematic change with U shows that a better fit is obtained by the introduction of an exponent that varies with T_g. This is of special importance when the variation of the hot-wire tension, V, with respect to turbulent fluctuations of both velocity and temperature has to be considered.

Table 2. Values of A, B and m at different gas temperatures and a wire temperature of 473 K for a platinated tungsten wire with $l = 1.93$ mm and $d = 2.5\,\mu$m

T_g (K)	T_f (K)	10^4A (A^2)	10^4B ($A^2 \cdot m^m \cdot s^{-m}$)	m	10^8AB ($A^4 \cdot m^{-m} \cdot s^m$)	10^4Bm ($A^2 \cdot m^{-m} \cdot s^m$)
283	378	7.30	8.30	0.393	60.6	3.26
293	383	7.51	8.02	0.408	60.2	3.27
303	388	7.87	7.67	0.421	60.4	3.23
313	393	7.93	7.57	0.423	60.0	3.20
333	403	8.61	6.89	0.457	59.3	3.15
353	413	8.91	6.64	0.474	59.2	3.15

Table 3. Values of A and B at $m = 0.40$ and $m = 0.45$ at gas temperatures of 293 K and 333 K and a wire temperature of 473 K

T_g (K)	T_f (K)	10^4A (A^2)	10^4B ($A^2 \cdot m^{-m} \cdot s^m$)	m	10^8AB ($A^4 \cdot m^{-m} \cdot s^m$)	10^4Bm ($A^2 \cdot m^{-m} \cdot s^m$)
293	383	8.42	7.08	0.40	59.6	2.82
333	403	8.45	7.04	0.40	59.5	2.82
293	383	7.30	8.23	0.45	60.1	3.70
333	403	7.24	8.27	0.45	59.8	3.72

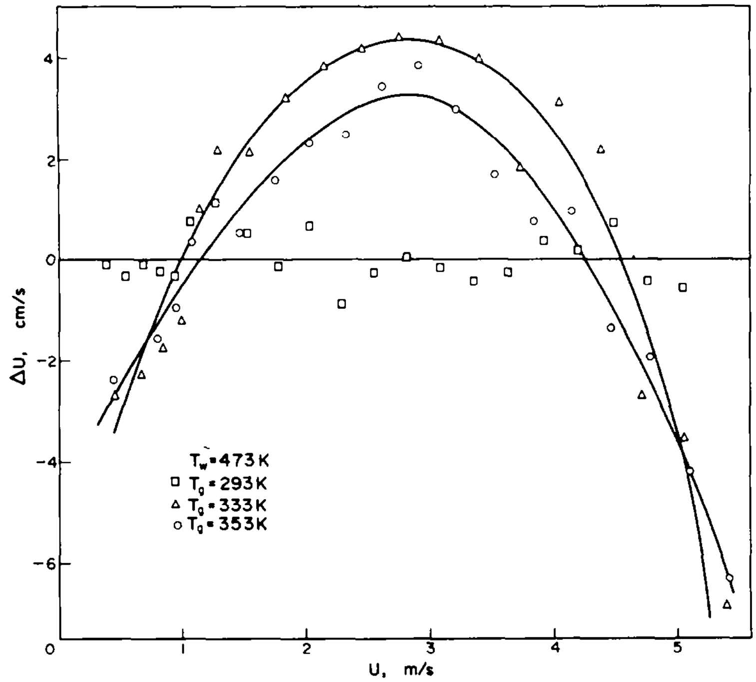


FIG. 4. Values of ΔU at different gas temperatures for $m = 0.40$.

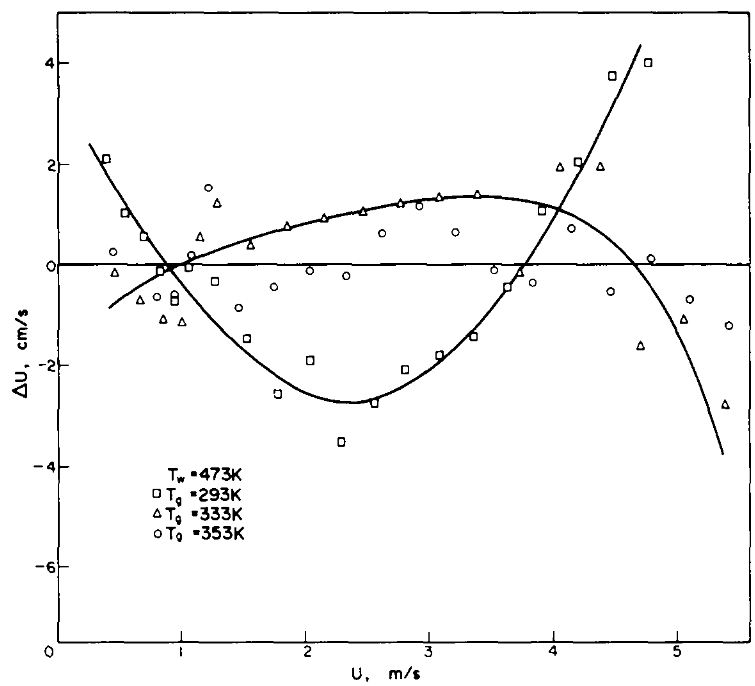


FIG. 5. Values of ΔU at different gas temperatures for $m = 0.45$.

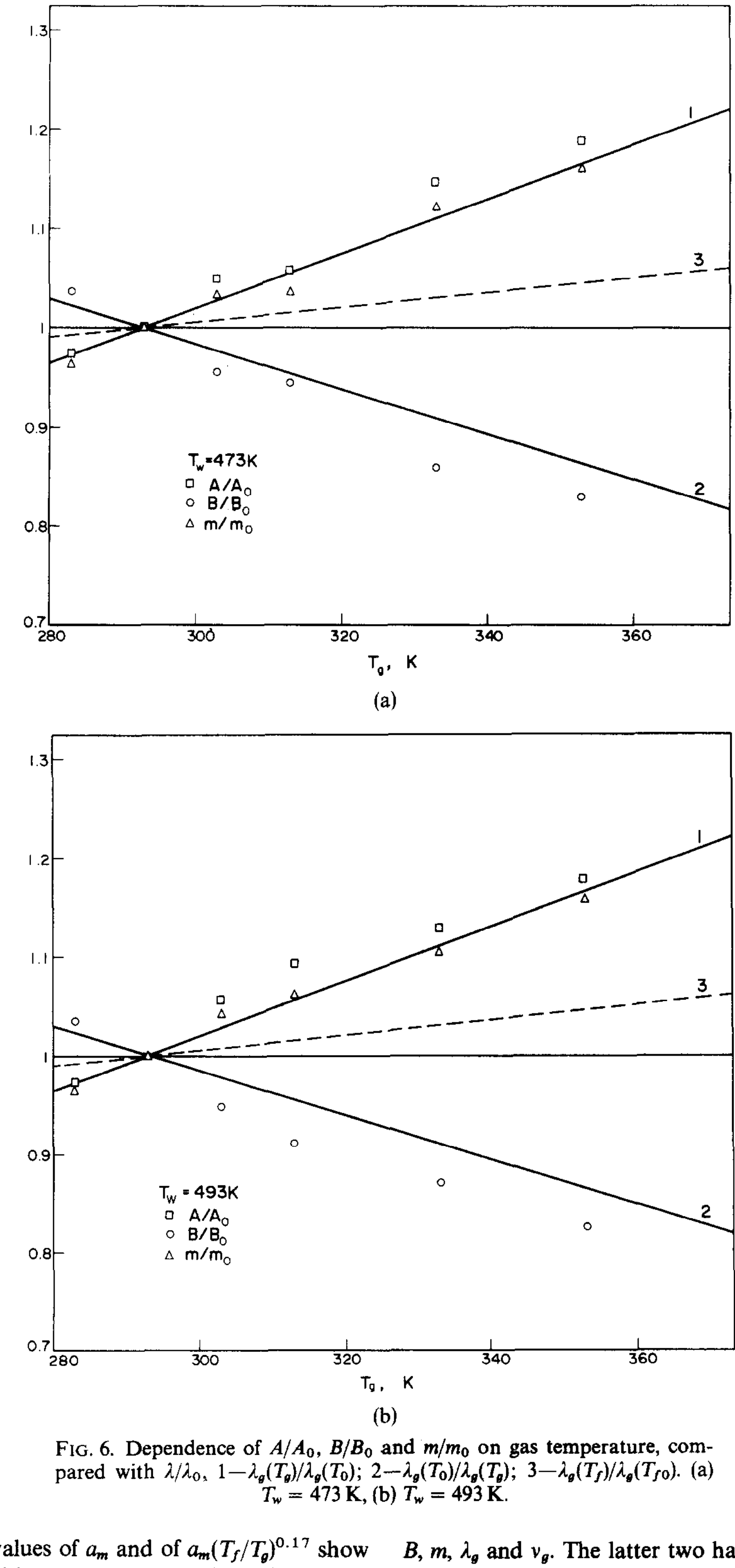
The temperature dependence of A, B and m is compared with that of $\lambda_g(T_g)$ in Fig. 6. It is seen that m varies proportionally with λ_g. The same holds approximately for A.

An even better proportionality between A/A_0 and λ_g/λ_{g0} is obtained when A is multiplied by the loading factor $(T_f/T_g)^{0.17}$ given by Collis and Williams [3]. B is found to vary in proportion with λ_g^{-1}. This, of course, is in agreement with the constancy of the products AB and mB. We therefore find empirically:

$$\frac{1}{A}\frac{\partial A}{\partial T} = \frac{1}{m}\frac{\partial m}{\partial T} = -\frac{1}{B}\frac{\partial B}{\partial T} = \frac{1}{\lambda_g}\frac{\partial \lambda_g}{\partial T} = \frac{0.814}{T} \quad (12)$$

where partial derivatives are taken at constant U and the temperature range is $283\,\mathrm{K} < T < 353\,\mathrm{K}$.

Values of a_m and b_m, calculated from equations (7) and (8) are given in Table 4, together with the values of these quantities multiplied by the loading factor



(a)

(b)

FIG. 6. Dependence of A/A_0, B/B_0 and m/m_0 on gas temperature, compared with λ/λ_0, 1—$\lambda_g(T_g)/\lambda_g(T_0)$; 2—$\lambda_g(T_0)/\lambda_g(T_g)$; 3—$\lambda_g(T_f)/\lambda_g(T_{f0})$. (a) $T_w = 473$ K, (b) $T_w = 493$ K.

$(T_f/T_g)^{0.17}$. The values of a_m and of $a_m(T_f/T_g)^{0.17}$ show little variation with temperature provided that in Nu the value of λ_g is taken at the gas temperature. This does not apply when λ_g is evaluated at the film temperature. This can also be seen from Fig. 6, since A/A_0 is proportional to $\lambda_g(T_g)/\lambda_g(T_0)$ but not to $\lambda_g(T_f)/\lambda_g(T_{f0})$. From equation (8) it follows that b_m depends on temperature in a complicated way through B, m, λ_g and ν_g. The latter two have been taken at T_g. However, the variation of b_m in Table 4 around its average value is restricted to $\pm 3\%$. This variation could be eliminated by the introduction of a second loading factor, $(T_f/T_g)^p$, with $p < 0$, taking into account the dependence of ν_g on temperature.

The values of Nu_m are non-continuum values in the sense of equation (2). Application of this equation leads

Table 4. Values of a_m, b_m and $a_m(T_g/T_f)^{0.17}$, $b_m(T_g/T_f)^{0.17}$ at different gas temperatures

T_g(K)	T_f(K)	a_m	b_m	$a_m(T_g/T_f)^{0.17}$	$b_m(T_g/T_f)^{0.17}$
283	378	0.313	0.700	0.298	0.666
293	383	0.312	0.689	0.298	0.658
303	388	0.311	0.679	0.298	0.651
313	393	0.312	0.669	0.300	0.644
333	403	0.322	0.665	0.312	0.631
353	413	0.317	0.647	0.309	0.630

to values of a, b and n that are somewhat higher than those of a_m, b_m and m respectively. However, the variation of these parameters with temperature is approximately the same in both cases. In interpreting these results it should be noted that equation (2) is based on a simplified theory of the non-continuum phenomena (cf. [11]) and that no pertinent information is available on the value of the thermal accommodation coefficient.

Recently an empirical correction for the dependence of the hot-wire calibration curve on temperature at low velocities was published by Kanevce and Oka [12]. Their temperature range (8.6–31.5°C) was narrower than ours and no correction was made for heat conduction to the supports. Therefore their results are not directly comparable with ours.

From a theoretical point of view and for practical purposes a temperature dependent exponent in equation (6) is awkward. Still it must be accepted in order to avoid systematic errors due to temperature variations. The alternative might be a more complicated relation expressing the dependence of Nu on Re, with more than three adjustable parameters.

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INFLUENCE DE LA TEMPERATURE DU GAZ SUR L'ETALONNAGE D'UN ANEMOMETRE A FIL CHAUD AUX FAIBLES NOMBRES DE REYNOLDS

Résumé—On a effectué des mesures de transfert de chaleur à l'aide d'un anémomètre à température constante pour des nombres de Reynolds 0,03 < Re < 0,80, le fil chaud étant placé perpendiculairement à un écoulement horizontal d'air. Les températures de gaz ont varié de 283 à 353°K, la température du fil chaud étant maintenue constante à 473°K. Après correction dûe à la conduction thermique par les supports, les résultats expérimentaux peuvent être représentés par la relation:

$$Nu_m = a_m + b_m Re^m$$

Tandis que a_m et b_m demeurent approximativement constants, on trouve que m varie avec la température du gaz de la même façon que sa conductivité thermique. La conductivité thermique et la viscosité cinématique du gaz sont prises à la température du gaz et non à la température du film, comme il est fait habituellement.

DIE ABHÄNGIGKEIT DER HITZDRAHTKALIBRIERUNG VON DER GASTEMPERATUR BEI KLEINEN REYNOLDSZAHLEN

Zusammenfassung—Messungen des Wärmeübergangs wurden an einem Hitzdraht als einem Konstant-Temperatur-Anemometer durchgeführt bei horizontaler Queranströmung mit Luft und 0,03 < Re < 0,80. Die Gastemperaturen wurden zwischen 283 K und 353 K variiert bei einer konstanten Temperatur des

Drahtes von 473 K. Nach Korrektur der Wärmeableitung in die Aufhängungen ließen sich die Versuchsergebnisse wiedergeben durch die Beziehung

$$Nu_m = a_m + b_m Re^m$$

Während a_m und b_m etwa konstant sind, zeigt sich für m eine Veränderung mit der Temperatur des Gases im gleichen Sinne wie die Aenderung der Wärmeleitfähigkeit des Gases. Wie praktisch üblich wurde die Wärmeleitfähigkeit und die kinematische Zähigkeit des Gases bei der Gastemperatur eingesetzt und nicht bei Filmtemperatur.

ЗАВИСИМОСТЬ ТЕРМОАНЕМОМЕТРИЧЕСКОЙ ТАРИРОВОЧНОЙ ХАРАКТЕРИСТИКИ ОТ ТЕМПЕРАТУРЫ ГАЗА ПРИ МАЛЫХ ЧИСЛАХ РЕЙНОЛЬДСА

Аннотация — Перенос тепла от нагретой нити, помещенной перпендикулярно к горизонтальному потоку воздуха, измерялся с помощью термоанемометра постоянной температуры при $0.03 < Re < 0.80$. Температура газа изменялась от 283 до 353°K при постоянной температуре нити 473°K. После поправки на потери тепла за счет теплопроводности ножек, к которым крепится нить, экспериментальные результаты можно представить следующей зависимостью:

$$Nu_m = a_m + b_m Re^m.$$

Найдено, что поскольку a_m и b_m остаются приблизительно постоянными, то m изменяется с температурой газа, как и его теплопроводность. Теплопроводность газа и кинематическая вязкость берутся при температуре газа, а не плёнки, как это обычно делается.